A Brief Tutorial on Endogeneity Bias and Instrumental Variables (IV) as a Possible Solution

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Exogenous vs. Endogenous Variables

- Exogenous variables are determined outside of the model, e.g., age, sex, race.
- Endogenous variables are determined within the model, i.e., as a simultaneous equations system.
  - e.g., consumer simultaneously chooses how much to spend on health care vs. other goods
- Endogenous variables expressed as functions of only exogenous variables are called “reduced-form” equations.

Endogeneity Bias

- Bias arises when one endogenous variable is regressed on another.
- Although endogeneity bias is not the same as correlation of the regressor with the error term (Greene), it is similar and the easiest way to think about this.
- If exogenous shocks affecting the outcome also affect any regressors, then $E(X'\varepsilon) \neq 0$ and you have a bias that IV methods might help with.

When Do You Have Endogeneity Bias?

Here are some questions to ask yourself about the regression you're running:

- Is the dependent variable determined simultaneously with any covariates?
- Can the story be told in both directions?
- Are any of the regressors correlated with the error term?

If the answer to any of these is yes, you may have endogeneity bias.

Examples of Potential Endogeneity

- Do women reduce work hours in response to informal caregiving?
- How much does insurance increase health care costs (price elasticity of demand)?
- Does having a usual source of care increase the use of preventive care?
- Does HMO membership reduce utilization?
- What is the “dose response” to substance abuse treatment?

Endogeneity Bias in Action

Even after adjusting for observable baseline severity...

- PIC: Depression treatment is associated with worse outcomes
- HCSUS: Medicaid insurance is associated with higher mortality

Both of these results reversed themselves when IV methods were used.

A Simple Example of How IV Works

Q: Does employment reduce depression among women?

Stylized Fact #1: Rates of depression are lower among women who are employed.

But…does this mean that employment lowers depression, or that women who are depressed are less likely to seek or obtain a job?

A Simple Example (cont’d)

Stylized Fact #2:

Women whose mothers worked during their formative years are also more likely to work themselves.

Stylized Fact #3:

Women whose mothers worked during their formative years have lower rates of depression.

A Simple Example (cont'd)

If maternal employment during the daughter's formative years does not *directly* influence whether the daughter is depressed later in life, then the *only* way to explain the lower rates of depression among daughters whose mothers worked is through the daughters' own employment: mother worked => daughter worked => daughter less depressed

A Simple Example (cont'd)

In other words, employment causally affects depression. The casual impact of employment on depression is "identified" through our assumptions that (1) the mother's employment (the "instrument") affects the daughter's employment but (2) does not directly affect the daughter's depression. Note that if either assumption fails, we cannot draw this causal inference.

A Formal Example of IV Methods

- What is the effect of informal caregiving on work hours?
- Structural equations are:

  $W = \delta I + \tau X_w + \varepsilon_W$ (1)

  $I = \lambda W + \gamma X_I + \varepsilon_I$ (2)

  We want a consistent estimate of δ.
- Treat both variables as continuous => special case of IV, known as two-stage least squares (2SLS).

Step 1: Run reduced-form regression and create predicted values for endogenous regressor

- Estimate the reduced-form regression of informal caregiving on all of the exogenous variables in the system:

  $I = \alpha X_W + \beta X_I + \eta$

- A predicted value is then constructed using the regression estimates:

  $I^P = \alpha^{hat} X_W + \beta^{hat} X_I$

Step 2: Estimate structural equation after replacing actual with predicted value for endogenous regressor

- Substitute the predicted value of informal caregiving from the 1st stage (I^P) for the actual value and then estimate:

  $W = \delta I^P + \tau X_w + \varepsilon_W$
- If doing this by hand, standard errors must be adjusted for use of predictions.

Intuition

- By using predicted rather than actual values of the endogenous regressor, you are “breaking” its correlation with the error term, so you get a consistent estimate of its structural effect, δ.
- The predicted values are just linear combinations of the exogenous variables, so by construction are not correlated with the error term.

Instruments and the "Exclusion" or "Identifying" Restriction

- Instruments for informal caregiving are variables included in X_I but not X_W.
- To "identify" the effect of I on W, we need at least one variable that affects I, but does not directly influence W after controlling for I and X_W. If X_I and X_W are identical, then I^P is collinear with X_W and its effect cannot be estimated.
- For example, parental health might be included in X_I but not X_W.

Identified Work Hours Equation

- For simplicity, assume that the only variables in X_I are the instruments.

$I = \alpha X_W + \beta X_I + \eta$ 1st stage regression

$W = \delta I^p + \tau X_w + \varepsilon_w$ 2nd stage regression

$= \delta(\alpha X_w + \beta X_i) + \tau X_w + \varepsilon_w$

$= (\delta\alpha + \tau)X_w + \delta\beta X_i + \varepsilon_w$

- We know α and β from the 1st stage and we can estimate $(\delta\alpha + \tau)$ and $\delta\beta$ by regressing W on X_w and X_I. Thus δ and τ can be uniquely identified.

Identified Work Hours Equation: Intuition

- To determine the effect of I on W, we want to measure how W changes when there is an exogenous shift in I, *holding everything else determining W constant*.
- Thus, we need to find something that will shift around I while the other regressors in the equation for W remain unchanged. Otherwise, we can't separate out the effect of I from the direct effects of the other regressors.

Unidentified Work Hours Equation #1

Case 1. X_i does not influence informal caregiving (weak instruments)

$I = \alpha X_w + \eta$

$W = \delta I^p + \tau X_w + \varepsilon_w$

$\quad = \delta(\alpha X_w) + \tau X_w + \varepsilon_w$

$\quad = (\delta\alpha + \tau)X_w + \varepsilon_w$

We know α from the 1st stage and we can estimate $(\delta\alpha + \tau)$ from a regression of W on X_w, but that isn't enough to uniquely identify δ and τ.

Unidentified Work Hours Equation #2

Case 2. X_i *has direct effects on both I and W (non-excludability)*

$I = \alpha X_w + \beta X_i + \eta$

$W = \delta I^p + \tau X_w + \theta X_i + \varepsilon_w$

$= \delta(\alpha X_w + \beta X_i) + \tau X_w + \theta X_i + \varepsilon_w$

$= (\delta\alpha + \tau) X_w + (\delta\beta + \theta) X_i + \varepsilon_w$

We know α and β from the 1st stage and we can get $(\delta\alpha + \tau)$ and $(\delta\beta + \theta)$ by regressing W on X_w and X_i, but since we don't know τ or θ, we can't figure out δ.

Interpretation

- For which respondents is δ a consistent estimate of the structural effect of I on W?
- At a minimum, this estimate applies to the "marginal" people, in this case, those for whom a change in the instrument (parental health) would change the endogenous regressor (informal caregiving).
- If the effect of I on W can be assumed to be homogeneous, then the estimate generalizes to the entire population.

Testing for Endogeneity: Hausman-Wu

- Under H_0 (no endogeneity): β_{OLS} and β_{IV} are both consistent estimators. However, β_{OLS} is efficient, while β_{IV} is inefficient.
- Under H_1 (endogeneity): β_{OLS} is inconsistent, while β_{IV} remains consistent.
- These circumstances fit the criteria for a Hausman-Wu test, so we can test H_0 using the χ^2 statistic:

$$(\beta_{IV} - \beta_{OLS})^T[VCV_{IV} - VCV_{OLS}]^{-1}(\beta_{IV} - \beta_{OLS})$$

Testing for Endogeneity: Augmented Regression

- Include both the predicted and actual values of the endogenous regressor in the 2nd-stage regression
- If the predicted value is statistically significant even when controlling for the actual value, then it is endogenous.
- The validity of endogeneity tests obviously depends on the validity of the instruments.

RCTs: An IV Application

- Suppose people are randomized to a treatment vs. control condition, but many do not end up getting as many services as the intervention called for
- “Intent-to-treat” design may understate the true effect of the services on HRQL
- We could use the random assignment to treatment condition as the instrument for HRQL in an “as-treated” analysis

RCTs: An IV Application (cont'd)

1st-stage regression:

Dose = αTreatment Assignment + δX + ε

2nd-stage regression:

HRQL = β*Predicted Dose + λX + η

Treatment assignment is assumed to influence HRQL only indirectly, through the dose of services received.

Main IV Assumptions

- *Non-zero average causal effect*   The instrument must predict the endogenous regressor, controlling for the other covariates.

- *Exclusion Restriction*   The instrument has only a negligible direct influence on the outcome after controlling for the covariates, that is, the instrument is uncorrelated with the error term.

Main IV Assumptions: A Note

- The key question is whether the correlation of the instrument with the endogenous regressor is high *relative* to its correlation with the dependent variable.
- The greater the problem of correlation with the dependent variable (failure of the exclusion restriction), the stronger its correlation with the endogenous regressor (non-zero average causal effect) needs to be in order to reduce bias.

Other IV Assumptions

- *Monotonicity* The instrument must have a monotonic effect on the endogenous regressor. For example, it cannot be the case that the instrument increases the value of the endogenous regressor for some subjects, but decreases it for others.
- *Random assignment* Subjects must be effectively randomized into the value for the instrument, at least within subgroups defined by the other covariates.

Other IV Assumptions (cont'd)

- *Stable unit treatment value assumption* The outcome of one subject is not influenced by the value of the endogenous regressor for other subjects.

  Depending on how you want to interpret the results, you must also assume that the effect of the endogenous regressor on the outcome is the same for all subjects.

IV Assumptions: An Example

- What is the effect of having a usual source of care (USC) on the use of preventive services?
- Having a USC is endogenous, because people who (unobservably) care about their health are simultaneously more likely to develop a relationship with a USC and to seek preventive care.
- Solution: IV estimation, using length of residence in the area as an instrument.

IV Assumptions: An Example (cont'd)

- *Non-zero average causal effect* Controlling for the other covariates, length of residence is a good predictor of having a USC.

- *Exclusion Restriction* Controlling for having a USC and the other covariates, length of residence does not predict use of preventive services.

IV Assumptions: An Example (cont’d)

- *Monotonicity* All subjects who would have had a USC if they lived in an area for a short time would also have one if they lived in the area for a long time.
- *Stable unit treatment value assumption* The use of preventive services by one subject is not affected by whether others have a USC, and differences among USC types in effectiveness are minor.

IV Assumptions: An Example (cont'd)

- *Random assignment* Subjects are effectively randomized into how long they have lived in an area within subgroups defined by the other covariates. That is, knowing a subject's use of preventive services does not yield any information about that subject's length of residence.

Testing the Instruments

- *Average non-zero causal effect*
  - Examine the magnitude and significance of the instruments in the 1st-stage regression, using an F-test of their joint significance.
- *Exclusion restriction*
  - χ^2 test, with the degrees of freedom equal to the number of overidentifying restrictions.
  - Can also perform sensitivity analyses, where you sequentially include one instrument at a time in the 2nd-stage regression.

The other assumptions require judgment.

Systems of Equations

- The procedure described above also holds if you are estimating a set of >2 equations. The 1st stage would involve estimating all the endogenous variables as functions of all of the exogenous variables.
- If you have >1 endogenous regressor, it is necessary but not always sufficient to have an equal number of instruments.
- Safe rule of thumb: have a unique instrument for every endogenous regressor

Summary

- Using single-equation estimation, the estimated effect of an endogenous regressor on the outcome will be biased and inconsistent, as will the effects of other regressors that are correlated with it (and in nonlinear models, *all* regressor effects, whether correlated or not)
- The direction of the bias is theoretically indeterminate if there is more than one covariate in the model, due to 2nd-order effects, i.e. correlations among regressors

Limitations of IV Analysis

- It’s difficult to find a valid instrument
  - Sometimes instruments are so bad that IV is actually more biased than OLS
  - Can’t test exclusion restriction unless you have multiple instruments, and even then, validity of test depends on having at least one good instrument
- You lose precision, especially if the instruments don’t predict the endogenous regressor very well

Places to Look for Instruments (borrowed from Staiger)

- Geography (distance, rivers, small area variation)
- Legal/political institutions (laws, election dynamics)
- Administrative/program rules (wage/staffing rules, reimbursement rules, eligibility rules, mandates)
- “Natural” randomization (draft, birthdate, lottery, roommate assignment, weather)